Parallel Session

Population Biology III

SENSITIVITY AND BIFURCATION ANALYSIS USING PREDATOR-PREY MODELS AS CASE STUDIES

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Keywords: Sensitivity analysis, Bifurcation analysis, Predator-prey model.

Model analysis methodologies like bifurcation analysis and sensitivity analysis each focus on particular model features and thus allow researchers to uncover only part of the model behaviour. By combining methodologies for model analysis possibilities arise for unravelling more of the ecosystem model's behaviour. Benefits and issues are discussed using a case study the Rosenzweig-MacArthur and the well-known Bazykin-Berezovskaya predator-prey model [1] where prey growth is not logistic but Allee function (hence cubic instead of quadratic). In the basic sensitivity analysis one parameter is varied and the consequences for model output are determined. When multiple parameters are changed separately, ranking of the resulting sensitivities gives their ranking of importance. The application is straightforward when the system is in stable equilibrium. Here we discuss also the more complex case where the attractor is a stable limit cycle [2, 3, 4]. Global sensitivity analysis methodologies aggregate samples (model output) from domains of attraction [5]. As a result, the qualitative difference in model behaviour between the domains of attraction is obscured. When multiple attractor exists this method has to be used with care, see [3]. Finally we discuss the structural sensitivity approach where model building blocks are exchanged, see [6, 7]. In [7] an extended Rosenzweig-MacArthur model called the Bazykin's model [1] with density-dependent mortality for the predator (hence quadratic instead of linear). In this paper two alternative functional response formulations are studied. The distance measure between the model formulation is a homotopy parameter which is one in the Holling type type II case and zero in the Ivlev functional response for the predator-prey consumption relationship. It was shown that a co-dimension-three Bogdanov-Takens bifurcation forms a separator in the bifurcation diagram. That is, the bifurcation portrait around the BT-point with the Holling type II formulation differs fundamentally from that with the Ivlev formulation.

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Parallel Session

Population Biology III

AN AGE-STRUCTURED MODEL WITH ALLEE EFFECT AND TIME DELAY

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Keywords: Age-structured model, Alee effect, Time delay, Asymptotical stability.

Impact of Allee effect on a nonlinear age-structured population dynamics with time delay is studied. Two female mating functions, negative exponential and rectangular hyperbolic are considered. The model assumes that death rate increases with the overall population size and it takes some time for individuals for mature.

We obtain conditions of existence of the trivial and up to two positive equilibria of autonomous system with constant coefficients for both female mating functions. The trivial equilibrium always exists and non-trivial equilibria exist if the basic reproduction number equals one. Stability of equilibria is studied first for simplified, autonomous model. It is proved that the trivial equilibrium is absolutely stable (i.e. locally asymptotically stable for all values of time-delay parameter – incubation period of newborn individuals) and the smaller of the two equilibria is unstable while the larger one is absolutely stable. If the model has only one positive equilibrium (which corresponds to the case where the characteristic value is zero), it is unstable without exponential growth in the vicinity of such an equilibrium. Secondly, for the non-simplified autonomous system with a narrower age reproductive window (the difference between the maximum and the minimum possible reproductive age of individuals) an explicit condition for local asymptotical stability of positive equilibria is derived in form of irrational equation. The interesting result of this study is that incubation period (i.e. delay parameter) and age reproductive window do not cause instability of model.

Several numerical experiments illustrate and confirm theoretical results.

MAXIMISING MICROBIAL GROWTH RATE IN CHANGING ENVIRONMENTS: THE μORAC FRAMEWORK

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Keywords: Metabolic pathways, Differential equations, Control theory, Microbiology.

Microbes such as bacteria and yeasts actively optimise their cellular growth rate by tuning concentrations of catalytic enzymes and ribosomes. They are able to switch metabolism to accommodate changes in food substrates, mount stress responses, and even shut down the entire cell and go in growth arrest when the need arises. In this talk we will present a general framework in which the allocation of enzymes and ribosomes necessary for cellular metabolism are adapted to steer the system to optimal steady state growth rate. The adaptive control uses only internal metabolic 'sensor' information, rather than direct knowledge of external concentrations. The cell can be made robust to changes in N environmental parameters if N sensor metabolites are employed. The framework allows for optimising growth rate, but also for simultaneously meeting stresses, such as cleaning up toxins or mounting a heat shock response. The μORAC framework therefore gives a unified rationale how cells can dynamically adapt their resource usage in order to maximise their fitness.

Parallel Session

Population Biology III

THE CHALLENGES OF MODELLING THE VAGINAL MICROBIOME

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Keywords: Bacteria, Population dynamics, Community ecology.

Clinical studies have shown that, unlike the gut microbiome, higher diversity in vaginal communities is associated with decreased stability and increased susceptibility to pathogen invasions. Notably, Lactobacillus-dominated communities are less diverse and are more effective at preventing infections. Several conjectures, pre-dating most microbiome studies, attempt to explain the stability of these vaginal microbial communities. We assess the viability of these conjectures using community ecology inspired theory. A common modelling approach in microbiome modelling is to use generalized Lotka-Volterra equations in which all pairwise interactions are free parameters to be inferred. However, microbial communities exhibit consumer-resource structure that imposes constraints on interactions. Here, we begin by explicitly including these constraints, grouping microbes, host cells and host bio-products into (resource/consumption) relevant groups. We compare low and high-dimensional models and fit them to longitudinal data of vaginal microbiome samples in order to disambiguate the conjectured mechanisms. In particular, we find that under most conditions resource-based interactions are stronger than microbe-microbe interactions (e.g. spite) and thus dynamical feedbacks between the host and microbes matter. This work echoes findings from ecology cautioning that ignoring resource-based interactions in our models of interacting species can lead to misleading conclusions and forecasts about community stability, species coexistence, and susceptibility to invasions.

Parallel Session

Population Biology III

STABILITY ANALYSIS OF GOMPERTZ'S LOGISTIC GROWTH EQUATION UNDER STRONG, WEAK AND NO ALLEE EFFECTS

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Keywords: Gompertz logistic growth equation, Stability analysis, Allee effects.

The interest and the relevance of the study of population dynamics and extinction phenomenon are the main motivation to investigate the induction of Allee effects in Gompertz's logistic growth equation. The stability analysis of the equilibrium points of Gompertz's logistic growth equation under strong, weak and no Allee effects is presented. Properties and sufficient conditions for the existence of strong, weak and no Allee effects for these new continuous population growth models are provided and discussed. It is established a sufficient condition to prove that the time evolution of the population density to the stable equilibria gets larger, as the Allee effects get stronger. These continuous population growth models subjected to Allee effects take longer time to reach its equilibrium states. The developed models are validated using the Icelandic herring population, with GPDD Id.1765.

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Parallel Session

Population Biology III

FROM GRADUAL CHANGES TO CATASTROPHIC SHIFTS IN ECOSYSTEMS WITH HUMAN INTERACTION

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Keywords: Ecosystems, Catastrophic Transitions, Bifurcation Theory, Numerical Analysis.

Many catastrophic transitions in ecosystems occur due to the interplay of both natural causes and human activities/interventions, as for example in the case of species extinctions or desertification. From a modelling and nonlinear dynamical point of view, ecosystems which undergo catastrophic transitions are often characterized by bistability which emanates from a turning point. Thus, the systematic modeling, analysis and forecasting of the complex behavior of ecosystems in response to their ongoing changes constitutes one of the major challenges of nowadays. In particular, more and more important is becoming the prediction of the effect of human interactions on ecosystems as political decisions that may be driven by social dynamics. Over the last few years, simple mathematical models in the form of ordinary and/or partial differential equations have been proposed to approximate in a qualitatively manner the observed complex phenomena. While, the vast majority of the studies dictate the importance of the notion of bifurcations for the better understanding of the mechanisms that pertain to the regime shifts, most of them use simple temporal simulations as a tool of choice for the system analysis. In this work, using the arsenal of numerical bifurcation theory, we analyze the impact of human interaction on an ecological model which represents the dynamics of a forest-grassland mosaic ecosystem. By constructing the bifurcation diagrams in the two dimensional parameter space with respect to the impact of human influence and natural causes, we were able to map the regions where different dynamics is observed and to detect codimension-2 bifurcations which mark the passage from a catastrophic shift to transitions characterized by smooth changes.